The Experimental Observation of a Superfluid Gyroscope in a dilute Bose Condensed Gas.

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We have observed a superfluid gyroscope effect in a dilute gas Bose-Einstein condensate. A condensate with a vortex possesses a single quantum of angular momentum and this causes the plane of oscillation of the scissors mode to precess around the vortex line. We have measured the precession rate of the scissors oscillation. From this we deduced the angular momentum associated with the vortex line and found a value close to $\hbar$ per particle, as predicted for a superfluid.

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The superfluid nature of a dilute gas Bose condensate, most strikingly demonstrated by its response to rotation, is currently an area of great experimental interest. The unique response of a superfluid to an applied torque arises because the presence of its single macroscopic wavefunction constrains the flow patterns allowed within its bulk. In [1], and later in [2,3], vortices with quantised circulation were observed when the trapping potential confining the atoms was rotated above a critical speed. Such vortices correspond to phase singularities within the gas where the density goes to zero. The quantised circulation (and hence angular momentum) associated with each vortex ensures that the superfluid wavefunction is single-valued at all points, whilst enabling the condensate to mimic the rotating flow pattern of a normal fluid. Secondly, the observation of the oscillation frequency of the 'scissors mode' [4] gave further evidence for superfluidity by showing that only an irrotational flow pattern (and not a rotational one) was possible in a vortex-free condensate.

In this work, we describe the realisation of a superfluid gyroscope as discussed by Stringari in [5], in an experiment that combines both vortex production and the excitation of the scissors mode. The scissors mode is a small-angle oscillation of the condensate relative to the trap potential, that is excited by a sudden tilt of the trap, as described in detail elsewhere [4,6]. In the gyroscope experiment we excite the scissors mode oscillation in the XZ or YZ plane of an axially symmetric condensate containing a vortex along the Z axis. In the presence of the vortex the plane of oscillation of the scissors mode precesses slowly around the Z axis. In polar coordinates, the scissors oscillation is in the θ direction and the precession is in the ϕ direction as shown in fig. 1. From the precession rate we deduce the angular momentum associated with the vortex line $\langle L_z\rangle$ and hence show that this angular momentum is quantised into units of $\hbar$ per particle, as predicted for a superfluid.

The relationship between the precession rate Ω and $\langle L_z\rangle$ may be derived by considering the scissors mode as an equal superposition of two counter-rotating $m = \pm 1$ modes [7]. These modes represent a condensate tilted by a small angle from the horizontal plane rotating around the z axis at the frequency of the scissors oscillation, $\omega_\pm = \omega_{sc}$. The symmetry and hence degeneracy of these modes is broken by the presence of axial angular momentum $\langle L_z\rangle$. Provided that the splitting is small compared to ω_{sc} we observe a precession of the scissors mode in good agreement with the theory of [5]:

$$\Omega = \frac{\omega_+ - \omega_-}{2} = \frac{\langle L_z\rangle}{2mN\langle x^2 + z^2\rangle} \tag{1}$$

where N is the total number of atoms in the condensate and m is the atomic mass. Substituting for $\langle x^2 + z^2\rangle$ in the case of a harmonically trapped condensate one obtains [5,8]

$$\Omega = \frac{7\omega_{sc}}{2}\frac{\langle L_z\rangle}{N\hbar}\frac{\lambda^{5/3}}{(1+\lambda^2)^{3/2}}\left(15N\frac{a}{a_{ho}}\right)^{-2/5} \tag{2}$$

where $\lambda = \omega_z/\omega_\perp$, $a_{ho} = (\hbar/m(\omega_\perp^2\omega_z)^{1/3})^{1/2}$ and $\omega_{sc} = (\omega_x^2 + \omega_z^2)^{1/2}$.

Other superfluid gyroscopic effects have been observed but the 3 - dimensional interaction between the velocity field of the vortex and the scissors mode of the condensate make this experiment unique. Superfluid gyroscopes of liquid helium exhibit persistent currents in toroidal geometries, with many quanta of circulation [9]. In contrast, we show here that a single vortex of angular momentum $\langle L_z\rangle = N\hbar$ significantly modifies the motion of a trapped BEC gas in an excited state. This is possible because the vortex produces relative shifts in the excitation spectrum of order ξ/R_0 and in such a dilute system the vortex core size ξ cannot be ignored with respect to the average size of the condensate R_0 [8]. Related experiments with vortex lines in dilute trapped gases are described in [10,11]. The angular momentum of a vortex line was measured in [10] using the precession of a radial breathing mode (a superposition of $m = \pm 2$ quadrupole modes) in the plane perpendicular to the vortex line. In

that work, motion is confined to 2 dimensions and the quadrupole oscillation of the condensate does not affect the vortex line. We observe the precession of a different quadrupole mode and the motion of the vortex line relative to the condensate is of interest. It has been suggested in [8] that the vortex line might follow the bulk motion of the condensate, and we present evidence consistent with that view. In [11] the precession of a vortex line is observed in the absence of any bulk condensate motion, when it is tilted or displaced from the condensate symmetry axes.

It is worth noting that a 'gyroscope' is a general term, describing a frictionless system with a large angular momentum vector that is able to rotate about any axis [12]. Our condensate, supported in a frictionless magnetic trap and with a vortex, is an example of such a system, although one must be careful not to draw incorrect analogies to classical gyroscope systems. Nutation, the wobbling motion superimposed on the precession of a spinning top is not analogous to the scissors motion in our system. The nutation frequency depends on the angular momentum, whereas the scissors frequency does not [13].

The first stage of exciting the superfluid gyroscope is to nucleate a single vortex at the centre of a condensate. A detailed discussion of the conditions for vortex nucleation in our apparatus is given in [3]. In summary the excitation procedure used for this experiment was as follows: First we produced a condensate in an axially symmetric TOP trap with $\omega_\perp/2\pi = 62\,\mathrm{Hz}$ and $\omega_z/2\pi = 175\,\mathrm{Hz}$. To spin up the condensate we made the trap eccentric ($\omega_x/\omega_y = 1.04$) over 0.2 seconds, with a trap rotation rate of 44 Hz. After holding the condensate in the spinning trap for a further 1 s we ceased the rotation of the trap potential by ramping both the trap rotation rate and the trap eccentricity to zero over 0.4 s. During the whole vortex excitation process, the RF evaporation was kept on, maintaining the temperature at approximately $0.5\,T_c$.

Figure 2 shows a perfectly centred vortex (a) and one that is at the edge of our criterion for an acceptably centred vortex (b). The position of the vortex line is important because we use destructive imaging; each data point requires a new condensate and identical starting conditions to give the same precession rate for each run. Equation 2 shows that the precession rate is affected by the number of atoms in the condensate and the angular momentum associated with a vortex line. Our shot-to-shot number variation of $N = 19000 \pm 4000$ produces a 10% variation in the precession rate. More significant is the number and position of vortex lines within the condensate since the precession depends linearly on $\langle L_z\rangle$. In a condensate of finite size, each vortex line is only associated with $\hbar$ of angular momentum per particle if it is exactly centred. The angular momentum associated with an off centre vortex in an axially-symmetric, harmonically trapped Thomas-Fermi condensate is [14,15]

$$\langle L_z\rangle = N\hbar\left(1 - \frac{d^2}{R^2{}_\perp}\right)^{5/2} \tag{3}$$

where d is the radial position of the vortex and $R_\perp$ is the radial condensate size. Using our second imaging system, that looks along the axis of rotation (z axis), we were able to check that $\sim 90\%$ of runs started with a single, clearly visible vortex positioned within a third of the condensate radius from the centre.

Immediately after making a vortex, the TOP trap was suddenly tilted to excite either the XZ or YZ scissors mode. A detailed description of the tilting procedure is given in [4] but in summary we apply an additional magnetic field to the TOP trap in the z direction, oscillating in phase with one of the radial TOP bias-field components, B_x or B_y, to excite either the XZ or YZ scissors mode respectively. The amplitude of this field was 0.55G, which combined with a 2G radial bias field tilts the trap by 4.4 degrees and hence excites a scissors oscillation of the same amplitude about the new tilted equilibrium position.

After allowing the oscillation to evolve for a variable time in the trap, we release it and destructively image along the y direction after 12 ms of expansion. By fitting a tilted parabolic density distribution to the image, we can extract the angle of the cloud and thus gradually build up a plot of the scissors oscillation as a function of evolution time. The visibility of the fast scissors oscillation depends on the angle of the cloud projected on the XZ plane (the plane perpendicular to the imaging direction) and hence varies at the slow precession frequency Ω. If the oscillation is in the XZ plane then the projected amplitude is maximum and if it is in the YZ plane then the projected amplitude is zero. By plotting the scissors oscillation as a function of evolution time, we observe the slowly oscillating visibility and hence extract the precession rate.

Figure 3 shows this plot when the scissors mode was originally excited in the XZ plane, perpendicular to the imaging direction. In (a) the condensate contained a vortex whilst (b) is a control run using condensates that did not contain a vortex. The fitting function used for each was

$$\theta = \theta_{eq} + \theta_0\,|\cos\Omega\mathrm{t}|\,(\cos\omega_{\mathrm{sc}}\mathrm{t})\,\mathrm{e}^{-\gamma\mathrm{t}} \tag{4}$$

with Ω set to zero for fig. 3b. In both cases the fast scissors oscillation is clearly visible and the fitted values of $\omega_{sc}/2\pi$ of (a) 179 Hz and (b) 186 Hz agree reasonably well with the theoretical value of 177 Hz. In the presence of a vortex the visibility shrinks rapidly to zero over 30 ms as the oscillation precesses through 90° to a plane containing the imaging direction. The oscillation visibility grows again after a further 90° precession. In the limit of small tilt angles the variation in oscillation visibility is represented by the $|\cos\Omega\mathrm{t}|$ term in eqn. 4. Note that $2\pi/\Omega$

is the time for a full 2π rotation and hence we expect the visibility to fall from maximum to zero in a quarter period, $\pi/2\Omega$. The fitted value of $\Omega/2\pi = 8.3 \pm 0.7\,\mathrm{Hz}$. From eqn. 2 this gives an angular momentum per particle, $\langle l_z \rangle$, of $1.14\ \hbar \pm 0.19\hbar$ for N = 19,000 ± 4000 atoms. In fig. 3a, each data point was taken 5 times and the mean and standard deviation is plotted. This averaging was necessary because the slight shot-to-shot variation in the starting conditions, produces slightly different precession rates.

The revived amplitude is smaller than the initial amplitude due to Landau damping, which occurs at a rate of $\gamma = 23 \pm 7\,\mathrm{Hz}$ from the exponential decay term in eqn. 4. Damping also occurs at a similar rate of $\gamma = 25 \pm 5\,\mathrm{Hz}$ in the control run, fig. 3b, without the presence of a vortex. Note that in (b) the condensate underwent the same spinning up procedure but at a trap rotation rate of 35 Hz, just too slow to create vortices. This ensured that in both cases the condensates were at the same temperature and hence had comparable Landau damping rates. The damping rates of approximately 24 Hz at a temperature of $0.5\,T_c$ agree well with the data about the temperature dependence of the scissors mode published in [16]. The control plot also confirmed that an axially symmetric condensate must have $L_z = 0$ unless a vortex line is present and hence the vortex is essential for precession.

Figure 4 shows the same experiment but with the scissors mode intially excited along the imaging direction so that the initial visibility is zero. The appropriate fitting function in this case is

$$\theta = \theta_{eq} + \theta_0 \left|\mathrm{sin}\Omega\mathrm{t}\right| (\mathrm{cos}\omega_{\mathrm{sc}}\mathrm{t})\, \mathrm{e}^{-\gamma\mathrm{t}} \qquad (5)$$

There was insufficient data to fit the Landau damping rate accurately in this case and so the value of γ was fixed at 24.2 Hz, as determined from the data of fig. 3. In the presence of a vortex (fig. 4a) the visibility of the scissors oscillation grows as the oscillation plane rotates through 90° to the XZ plane. This **growth** of an oscillation is perhaps a more significant proof of precession than the initial decrease of amplitude in fig. 3a, since it cannot be explained by any damping effect. The precession rate from fig. 4a is $7.2 \pm 0.6\,\mathrm{Hz}$, which agrees within the stated errors with the precession rate of fig. 3a and gives $\langle l_z \rangle = 0.99\hbar \pm 0.17\hbar$. In fig. 4b there was no vortex and so the oscillation remained in the YZ plane, with zero angle projected onto the XZ direction.

Note that the mean angles in fig. 3a and fig. 4a are different. This mean angle corresponds to the trap angle (the cloud angle in equilibrium) in the visible XZ plane. In fig. 3 the trap tilt occurs in the XZ plane and so this mean angle is θ_{eq}, whereas in fig. 4 the tilt is in the YZ plane, and so the mean angle in the imaging plane is zero.

Combining the results for the XZ and YZ gyroscope experiments, we measure the angular momentum per particle associated with a vortex line to be $1.07\hbar \pm 0.18\hbar$. This is in excellent agreement with the value of $\hbar$ per particle predicted by quantum mechanics. It is interesting to note that we deduce an angular momentum per particle slightly greater than $\hbar$, even though only one vortex is visible. Given that this vortex is not always perfectly centred, one might expect to observe a value for Ω and hence $\langle L_z \rangle$ that is slightly lower than the theoretical value.

One explanation is that under conditions which *reliably* produce a single, centred vortex, vortices are also created at the edge of the cloud, which make a small additional contribution to the angular momentum (eqn. 3) but are not observable. This explanation is in agreement with the results of Chevy *et al.* in [10].

Finally we discuss the motion of the vortex core, prompted by the suggestion in [5] that it might exactly follow the axis of the condensate. Images taken perpendicular to the vortex core were necessary to show the angle of the core relative to the axis of the condensate. However in a dense condensate the vortex core is too small to have a discernable effect on the integrated absorption profile. Thus we reduced the number of atoms in the condensate to $\leq 10,000$ to obtain the images in fig. 5. Vortices were clearly visible on 50% of the experimental runs and within the limits of the imaging system resolution and pixel to pixel noise, it appears that the vortex line is aligned with the condensate axis in 95% of these shots.

In conclusion, we have observed a superfluid gyroscope effect in a trapped BEC, in which a single quantum of circulation (a single vortex line) affects the bulk scissors motion of the condensate and causes it to precess. By observing the precession rate we are able to measure the angular momentum associated with the vortex line. Our result of $1.07(\pm 0.18)N\hbar$ agrees well with $N\hbar$, the quantum of angular momentum that is predicted to be associated with each centred vortex line.

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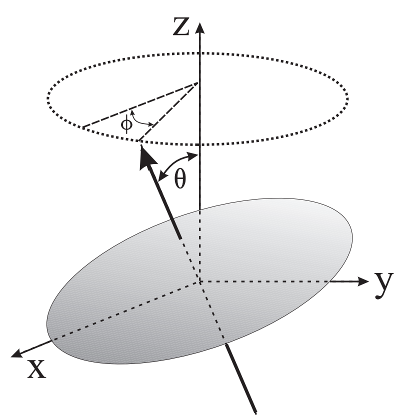

FIG. 1. The gyroscope motion. The condensate performs a fast scissors oscillation in the θ direction at ω_{sc}, whilst the plane of this oscillation slowly precesses in the ϕ direction at frequency Ω. The vortex core moves independently and is not shown on this diagram.

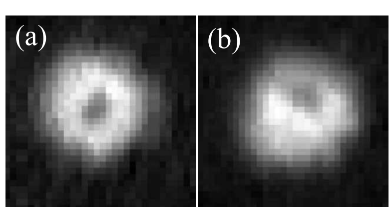

FIG. 2. Expanded images of vortices prior to excitation of the scissors mode, viewed along the axis of rotation. (a) shows a single centred vortex whilst (b) shows a vortex at the edge of our criterion for an 'acceptably centred vortex'.

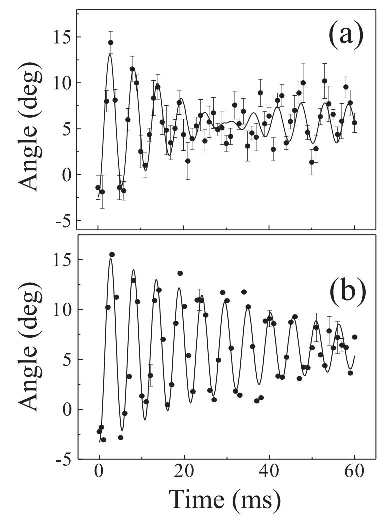

FIG. 3. The angle of the cloud projected on the XZ plane when the scissors mode is initially excited in the XZ plane, in (a) with a vortex and in (b) without a vortex. In (a) each data point is the mean of 5 runs, with the standard error on each point shown. The solid line is the fitted function given in eqn. 4. In (b) most data points are an average of 2 runs, occasionally 5 runs were taken and the standard error is shown for these points for comparison with (a).

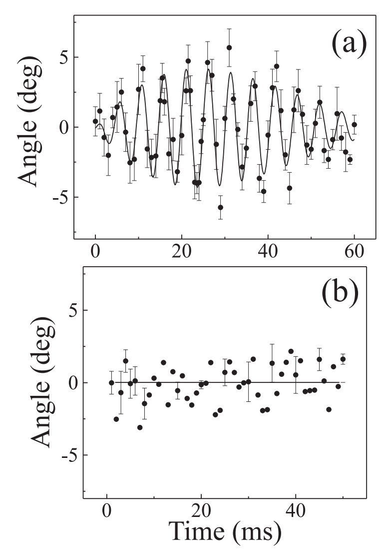

FIG. 4. As for fig. 3 but with the scissors mode initially excited in the YZ plane, and eqn. 5 as the fitting function (solid line).

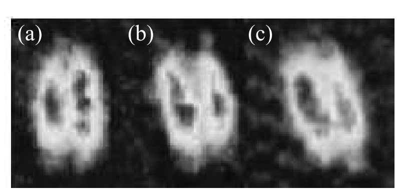

FIG. 5. Absorption images taken along the y direction after 12 ms of expansion show the vortex core following the angle of the axis of the condensate during the gyroscope motion. These images were taken with a smaller condensate density ($N \leq 10,000$), so that the vortex core (of radius $(8\pi na)^{-1/2}$) was large enough to have a significant effect on the integrated absorption profile.